A36932

Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers



UNIVERSITY OF BIRMINGHAM

School of Computer Science

Second Year Undergraduate

06-34255

LI Artificial Intelligence 2

Main Summer Examinations 2026

Time allowed: 2 hours

[Answer all questions]

Note

Answer ALL questions.

Question 1 [Bayes Networks and Classifiers - 42 marks]

A robotics lab deploys an autonomous warehouse robot. A safety system must decide whether to **halt** the robot ($Y = 1$) or **continue** ($Y = 0$) based on three binary sensor flags:

$$X_1 = \text{Overheat}, \quad X_2 = \text{Obstacle proximity}, \quad X_3 = \text{Motor vibration}.$$

The lab proposes a Bayesian network:

$$Y \to X_1, \quad Y \to X_2, \quad (Y, X_2) \to X_3$$

i.e. X_3 depends on Y and X_2.

They estimate the parameters:

$$P(Y = 1) = 0.2.$$

$$P(X_1 = 1 \mid Y = 1) = 0.7, \quad P(X_1 = 1 \mid Y = 0) = 0.1.$$

$$P(X_2 = 1 \mid Y = 1) = 0.6, \quad P(X_2 = 1 \mid Y = 0) = 0.2.$$

$P(X_3 = 1 \mid Y, X_2) =$

	$X_2 = 0$	$X_2 = 1$
$Y = 0$	0.05	0.30
$Y = 1$	0.40	0.90

(a) Answer the following:

(i) Write the joint factorisation $P(Y, X_1, X_2, X_3)$ implied by the DAG. **[4 marks]**

(ii) Determine whether the following conditional independences hold. Justify your answers.

i. $X_1 \perp X_2 \mid Y$ **[2 marks]**

ii. $X_1 \perp X_3 \mid Y$ **[2 marks]**

iii. $X_1 \perp X_3 \mid (Y, X_2)$ **[2 marks]**

(b) Compute the following posterior probabilities:

(i) $P(Y = 1 \mid X_1 = 1, X_2 = 1, X_3 = 1)$. **[5 marks]**

(ii) $P(Y = 1 \mid X_1 = 1, X_2 = 0, X_3 = 1)$. **[5 marks]**

(c) Another lab considers Naive Bayes with class Y and features X_1, X_2, X_3 instead of the Bayes net, incorrectly assuming

$$X_1 \perp X_2 \perp X_3 \mid Y.$$

(i) Compute $P_{\text{NB}}(X_3 = 1 \mid Y = 1)$ and $P_{\text{NB}}(X_3 = 1 \mid Y = 0)$ implied by the Bayes net parameters (i.e. by marginalizing out X_2). Here, the subscript NB refers to probability estimates computed according to the Naive Bayes model. **[6 marks]**

(ii) Compute the naive Bayes posterior $P_{\text{NB}}(Y = 1 \mid X_1 = 1, X_2 = 1, X_3 = 1)$. Compare it to the exact posterior from (b)(i) and discuss which model is more confident in its prediction. **[4 marks]**

(d) The lab also trains a depth-1 decision tree (a stump) on a large dataset whose distribution matches the Bayes net above.

(i) In preparation to building the decision tree, compute $P(Y = 1 \mid X_1 = 1)$ and $P(Y = 1 \mid X_2 = 1)$. **[6 marks]**

(ii) Compute the *information gain* at the root for splitting on X_1 and on X_2 (use natural logs). Indicate which split is chosen. **[6 marks]**

Question 2 [Certification of Learning - 25 marks]

(a) Use the corollary to the Test Set Bound to answer the following:

(i) Give an upper bound on the true generalisation error of any *fixed* classifier that achieves an error of 0.01 on the test sample with probability at least $1 - \delta$. **[5 marks]**

(ii) Suppose for a fixed classifier we observe 0.01 test error. How large does the test set need to be to guarantee with confidence 99% that the true generalisation error is no more than 0.02? Use natural logs, and show your working. **[5 marks]**

(b) A team ran a model-selection tournament. They trained M different classifiers on the same training set. They then evaluated *all* M models on the same held-out test set of size n and reported the best-performing model.

Let $\hat{R}_j$ denote the test error of model j, and suppose the reported winner has a test error $\min_j \hat{R}_j = 0.01$.

(i) Derive a guarantee that holds for *all* M models simultaneously, and hence for the selected winner. **[5 marks]**

Hint: Use a union bound.

(ii) Suppose $M = 500$ models were tried and the team wants a guarantee a true error of $R_{\text{winner}} \leq 0.02$ with confidence 99%. Find the minimum test set size n required for this. Use natural logs, and show your working. **[5 marks]**

(c) Can you come up with a mechanism by which the winner can achieve $\hat{R}_{\text{min}} = 0$ while having true risk close to $1/2$ on future data, even if n is moderate? If yes, describe how. If no, explain why not. **[5 marks]**

Question 3 [Constraint Satisfaction Problem (CSP) - 33 marks]

(a) Give at least two differences between tree search and local search when solving constraint satisfaction problems. **[10 marks]**

(b) Mr. Smith has two daughters, Girl1 and Girl2. The younger daughter is 8 years old, and the older daughter is 14 years old. The two girls like different colours from {blue, green, red}, and different pets from {cat, dog, fish, rabbit}. We know the following facts:

- Girl1 does not like fish.
- The older girl likes cat.
- The girl who likes blue also likes fish.
- One of the girls likes red.

Our task is to determine the ages of Girl1 and Girl2, as well as the colour and pet each of them likes.

(i) Formulate this problem as a CSP problem. Specifically, you need to give the variables, domains and constraints. You may use variables $Age1$, $Colour1$ and $Pet1$ to represent the first girl's age, the colour she likes, and pet she likes, respectively. All constraints should be specified formally (i.e., using these variables). **[10 marks]**

(ii) Draw the constraint graph associated with your CSP. **[4 marks]**

(iii) Use backtracking search with forward checking and ordering to find a solution to this problem. Assume that ties of variables are broken alphabetically and then numerically (i.e., in the order: $Age1$, $Age2$, $Colour1$, $Colour2$, $Pet1$, $Pet2$). The value of a variable should be considered in alphabetical or numerical order as appropriate (i.e., 8, 14 for the age variables; blue, green, red for the colour variables; and cat, dog, fish, rabbit for the pet variables). List the order of states visited during the search. **[9 marks]**

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.