Exercise Sheet 1

Neural Computation
University of Birmingham

Week 1 2025

Exercise 1. Consider the following perceptron $\mathbf{w} = (1, 1, 3)$, $b = 2$. Compute the output $\hat{y}$ of the perceptron for the following points:

(a) $\mathbf{x} = (0, 0, 1)$

(b) $\mathbf{x} = (0, 0, 0)$

(c) $\mathbf{x} = (-1, -1, -3)$

(d) $\mathbf{x} = (-2, -2, , 0)$

Solution:

We are given a perceptron with weight vector

$$\mathbf{w} = (1, 1, 3), \quad b = 2.$$

The perceptron computes

$$\hat{y} = \begin{cases} 1, & \text{if } \mathbf{w} \cdot \mathbf{x} + b > 0, \\ -1, & \text{otherwise.} \end{cases}$$

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a) For $\mathbf{x} = (0, 0, 1)$:

$$\mathbf{w} \cdot \mathbf{x} + b = (1)(0) + (1)(0) + (3)(1) + 2 = 3 + 2 = 5 > 0.$$

Thus, $\hat{y} = 1$.

b) For $\mathbf{x} = (0, 0, 0)$:

$$\mathbf{w} \cdot \mathbf{x} + b = (1)(0) + (1)(0) + (3)(0) + 2 = 2 > 0.$$

Thus, $\hat{y} = 1$.

c) For $\mathbf{x} = (-1, -1, -3)$:

$$\mathbf{w} \cdot \mathbf{x} + b = (1)(-1) + (1)(-1) + (3)(-3) + 2 = -1 - 1 - 9 + 2 = -9 < 0.$$

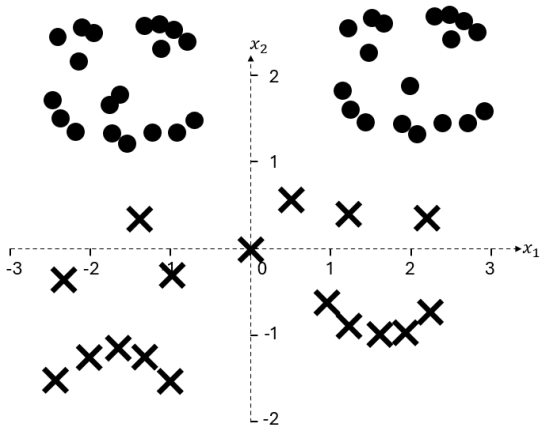
Thus, $\hat{y} = -1$.

d) For $\mathbf{x} = (-2, -2, 0)$:

$$\mathbf{w} \cdot \mathbf{x} + b = (1)(-2) + (1)(-2) + (3)(0) + 2 = -2 - 2 + 0 + 2 = -2 < 0.$$

Thus, $\hat{y} = -1$.

Exercise 2. Consider the following datapoints:

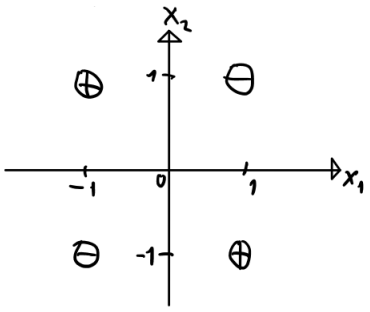


Is it possible to find a weight vector $\mathbf{w}$ and bias b, so that the resulting perceptron fully separates circles from crosses.

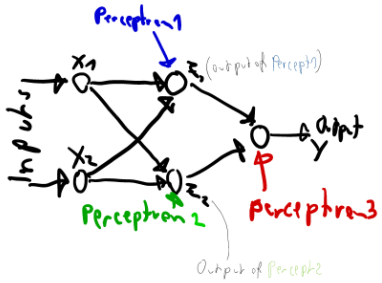
Solution:

We could use $\mathbf{w} = (0, 1)$ and $b = -1$. This would make the hyperplane a horizontal line at $x_2 = 1$ Note that b has to be negative to shift the hyperplane into the direction of $\mathbf{w}$.

Exercise 3. Consider the following data points in part A) of the Figure below. It is not possible to separate the datapoints indicated by '+' from the ones indicated by '-' minus using a simple perceptron. However, it is possible to separate them using 3 perceptrons as shown in part B), where the two inputs x_1, x_2 both go into perceptrons 1 and 2, and the outputs z_1, z_2 form both of them are combined in perceptron 3 to produce a final classification.



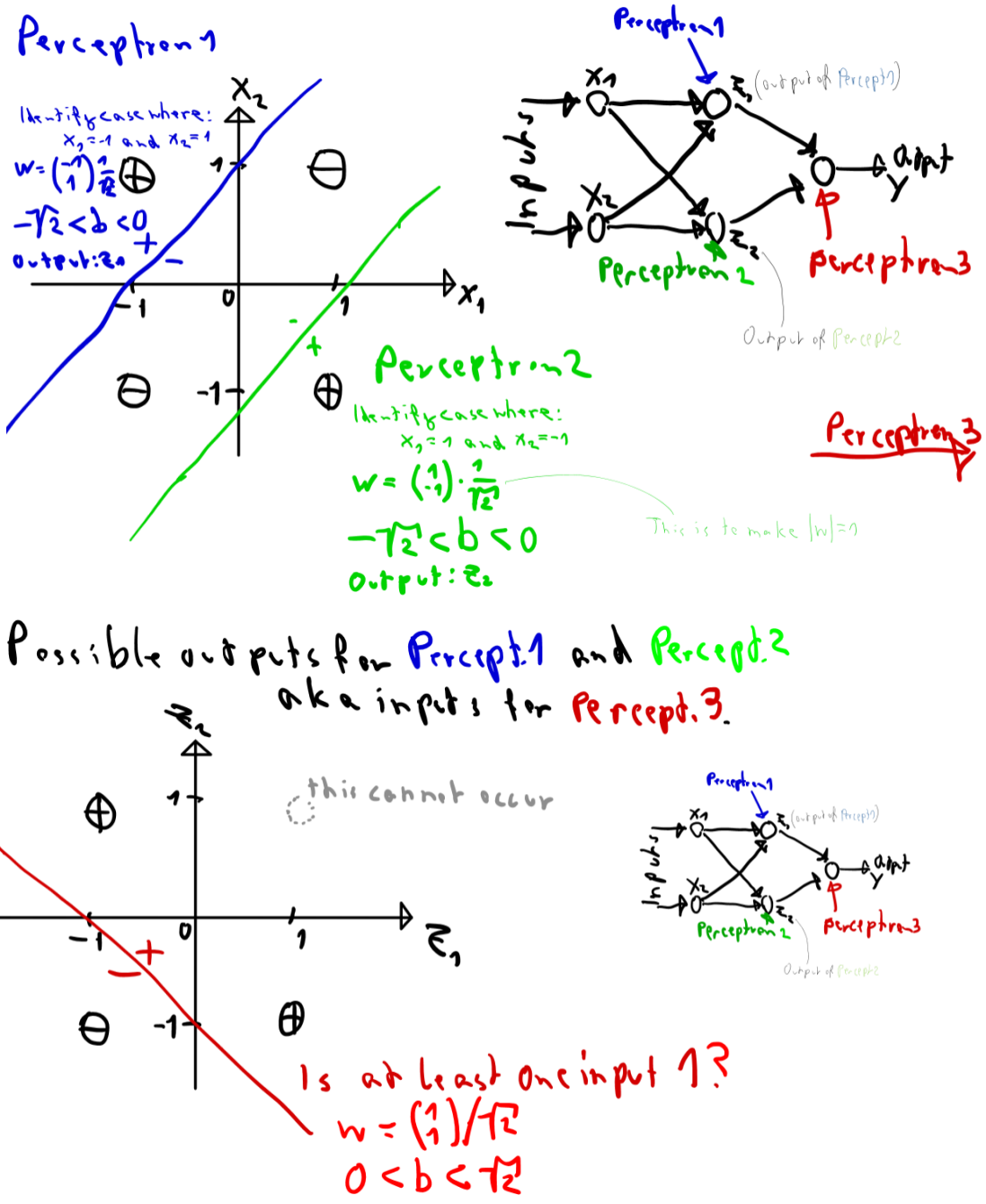
A)



B)

Find a weight vectors $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$ and biases b_1, b_2, b_3, so that the resulting arrangement of perceptrons fully separates '+' datapoints from '-' datapoints.

Solution:



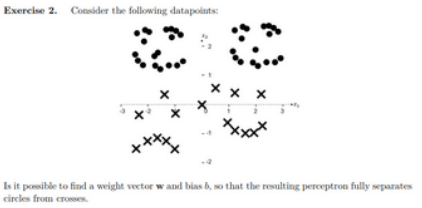
Exercise 4. (Bonus) Take a screen shot of Excercise 2 (including the text above and below the plot) and try to solve it using a GenAI chatbot tool like ChatGPT or Gemini. Can you get the correct solution?

Solution:

The chatbots usually insist that this is not possible. Persumingly because the question implies that it may not be possible they confuse the question with a situation like the one in Excercise 3, which is a classical example of non-linearly separable data used in many text books and courses. This is a nillustration that these chatbots currently do not think 'logically' like we humans do.

You are likely to get the correct answer by just pasting the plot itself and asking it to find the solution, without implying that this may not be possible.

Below is the output that Gimini gave us. Copilot and ChatGPT gave similar answers:



Solve this

Analysis

Based on the provided image and the nature of a perceptron, here is the solution.

The answer is **No**, it is **not possible** to find a weight vector $\mathbf{w}$ and bias b that allows a resulting **perceptron** to fully separate the circles (positive class) from the crosses (negative class).

Perceptron's Limitation

A perceptron is a model for a **linear classifier**. This means it can only separate data points that are **linearly separable**.

- **Linearly Separable Data:** Data is linearly separable if a single straight line (or a hyperplane in higher dimensions) can be drawn to divide the two classes, placing all points of one class on one side and all points of the other class on the opposite side.
- **Perceptron's Decision Boundary:** The decision boundary of a perceptron is defined by the

Ask Gemini

Tools