Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

UNIVERSITY OF BIRMINGHAM

School of Computer Science

Advanced Robotics

Main Summer Examinations 2024

Time allowed: 2 hours

[Answer all questions]

Turn Over

Note

Answer ALL questions. Each question will be marked out of 20. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Question 1

(a) For the set of minimal representations of orientation ϕ, consider the sum operation in terms of the composition of rotations. Using an example, show that the commutative property does not hold for that operation. You are given the matrix $R(\phi)$ which is obtained by the composition of elementary rotations around ZYZ. **[10 marks]**

$$\begin{aligned}\boldsymbol{R}(\boldsymbol{\phi}) &= \boldsymbol{R}_z(\varphi)\boldsymbol{R}_{y'}(\vartheta)\boldsymbol{R}_{z''}(\psi) \\ &= \begin{bmatrix} c_\varphi c_\vartheta c_\psi - s_\varphi s_\psi & -c_\varphi c_\vartheta s_\psi - s_\varphi c_\psi & c_\varphi s_\vartheta \\ s_\varphi c_\vartheta c_\psi + c_\varphi s_\psi & -s_\varphi c_\vartheta s_\psi + c_\varphi c_\psi & s_\varphi s_\vartheta \\ -s_\vartheta c_\psi & s_\vartheta s_\psi & c_\vartheta \end{bmatrix}\end{aligned}$$

(b) Draw a kinematic schematic for the robot which is described by DH parameters in Table 1. In your drawing use cylinders to illustrate revolute joints and cubes for prismatic joints. Clearly label all parameters and frame axes. Note that a_1, a_2 and a_3 are constant parameters. **[10 marks]**

Table 1: DH parameters for a robot in Question 1

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	0	θ_1
2	0	0	0	θ_2
3	l_3	$-\pi/2$	d_3	$\pi/2$
4	0	0	l_4	θ_4

Question 2

(a) Consider a 3DoF RRP manipulator robot which is shown in Figure 1. Compute the position and the velocity vectors of the end-effector with respect to the base frame. **[10 marks]**

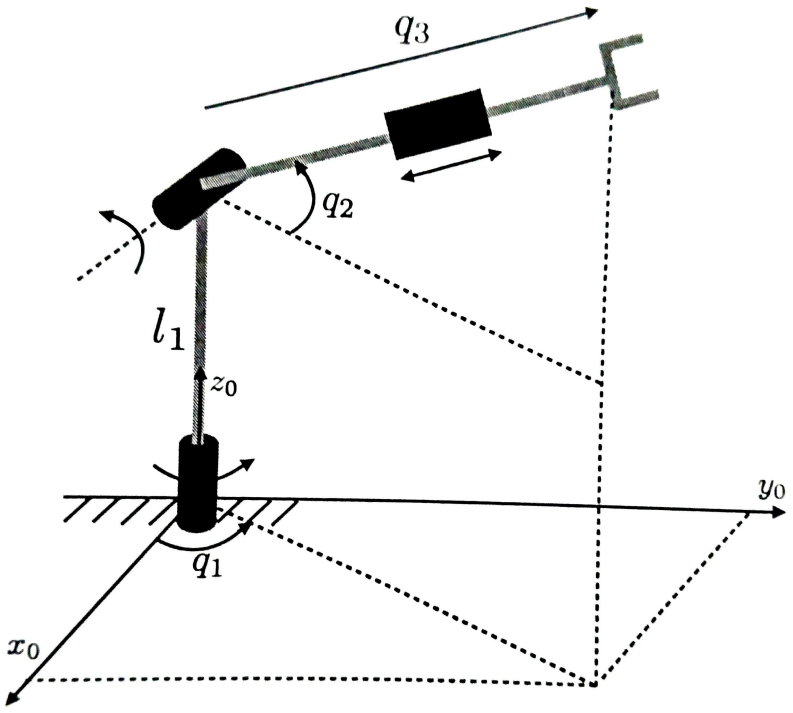


Figure 1: 3DoF RRP manipulator robot

(b) Consider a minimal representation of the orientation of a rigid body by Euler angles as $\phi = (\alpha, \beta, \gamma)$. Note that α, β, and γ are rotation angles about current x, y, and z axes, respectively. Determine the relationship between angular velocities and time derivatives of the Euler angles. **[10 marks]**

$$\omega = \boldsymbol{T}(\phi)\dot{\phi}$$

Question 3

You are asked to develop a planner for two mobile robots in an office, as shown in Fig. 2. The robots' tasks are focused on navigation. The robots navigate over a topological map $\mathcal{T} = \langle V, E \rangle$, where V is a set of nodes, and E is a set of undirected edges (see Fig. 2). All edges have the same length and can be navigated in one timestep. If the two robots navigate on different edges, their respective navigation actions succeed with probability 0.8. If they navigate on the same undirected edge at the same time, their

respective success probabilities drop to 0.6. If navigation fails, the robot remains in its current position. The planner should minimise the sum of durations. After each action, the robots accurately observe the position of all robots.

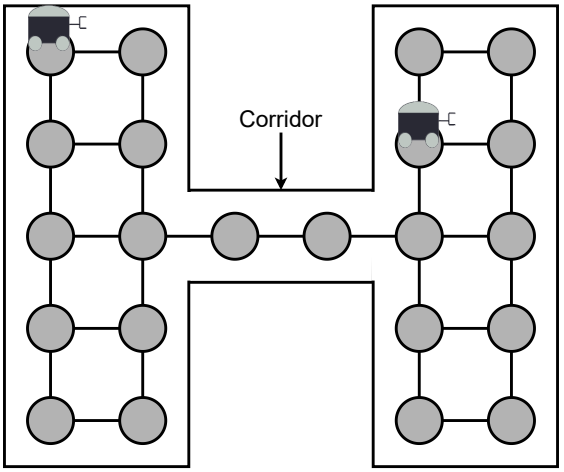


Figure 2: Two robots navigating in an office. The grey circles are topological nodes, and the lines connecting them are the undirected topological edges.

(a) Which modelling formalism is suitable for this problem and why? **[2 marks]**

(b) Using your chosen modelling formalism, provide a mathematical definition of the planning model for the robots in Fig. 2, and briefly describe each component. **[11 marks]**

(c) What is the primary challenge when planning with your chosen modelling formalism, and why? **[2 marks]**

(d) The robots have recently been equipped with a state of the art motion planner which reduces inter-robot collisions. As a result, their interactions primarily impact each other's navigation success probabilities when they simultaneously navigate through the corridor depicted in Figure 2. Explain which modelling formalism you would employ and justify your choice. **[3 marks]**

(e) Each individual robot is equipped with a basic version of sampling based motion planning method to navigate from one state to another. Which sampling based motion planning is suitable for this domain? Justify your answer. **[2 marks]**

End of Paper

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.