A39874

Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

UNIVERSITY OF BIRMINGHAM

School of Computer Science

Fourth Year Undergraduate/Postgraduate

06-40096

LM Robot Motion Planning and Control

Main Summer Examinations 2025

Time allowed: 2 hours

[Answer all questions]

Note

Answer ALL questions.

Question 1

(a) The rotation matrix that is obtained by the composition of ZYZ elementary rotations α, β and γ is shown below. Using this matrix, show why we cannot determine each rotation uniquely. In other words, why does the representation of rotations as Euler angles suffer from singularity? **[13 marks]**

$$R = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix} \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) We want to design a robot that is mainly employed for machining. Due to the constraints of the environment, the workspace of the desired robot is shown in Fig. 1. This robot has 3 joints.

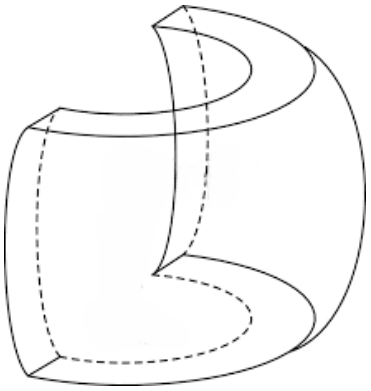


Figure 1:

(i) Draw a kinematic schematic of the robot described above. **[6 marks]**

(ii) Assign proper frames to each link of the manipulator and derive the table of Denavit-Hartenberg (DH) parameters. **[14 marks]**

Question 2

(a) Consider the 3DoF PRP manipulator robot which is shown in Figure 2. Using the analytical Jacobian of the robot's end-effector, find configurations at which the mobility of this manipulator is reduced, i.e., it is not possible to impose an arbitrary motion to the end-effector. **[18 marks]**

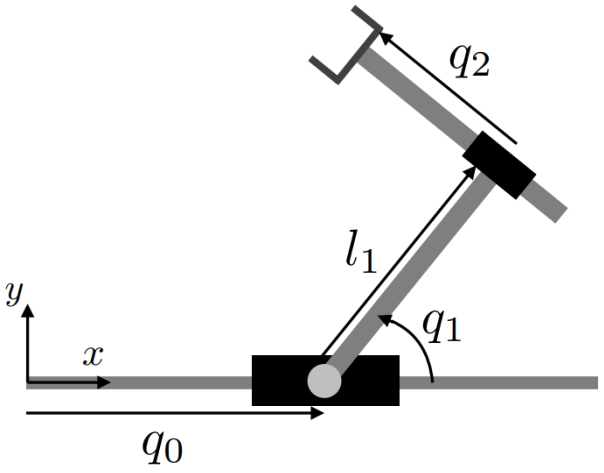


Figure 2: 3DoF PRP manipulator robot

(b) Assume the Jacobian matrix J is full rank and let J^+ be its Moore-Penrose pseudoinverse. Given the inverse kinematics equation below

$$\dot{q} = J^+ \dot{x}_d + (I - J^+ J)\dot{q}_0$$

show that $(I - J^+ J)$ is a null space projection matrix. **[14 marks]**

Question 3

Robot 1 that is shown in Fig. 3 is a disk-shaped robot of radius 1 m. Robot 1 operates in a warehouse whose 2-D map is also shown in Fig. 3 with obstacles shaded black. You are to design a planner that reasons about free space using a Voronoi graph.

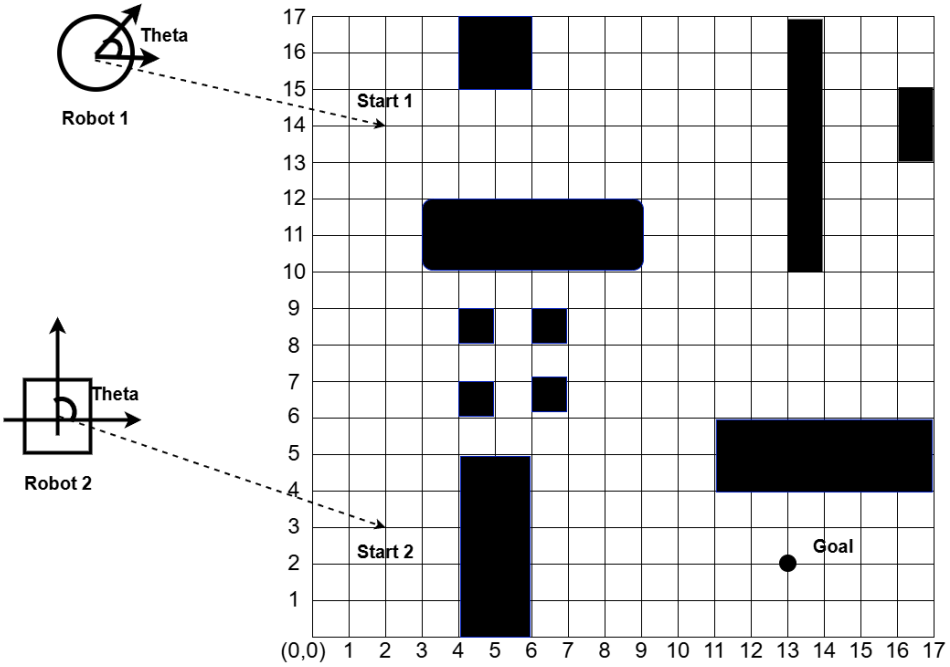


Figure 3: 2D map of a warehouse environment with two robots.

(a) Draw the Voronoi graph on the 2-D map shown in Fig. 3, and using the graph, show the shortest path from start 1 to goal position on the map. **[14 marks]**

(b) Explain the challenges of using the planner that operates on a Voronoi Graph. **[4 marks]**

(c) Robot 2, shown in the bottom left corner of Fig. 3, is a car-like mobile robot with a square shape, each side measuring 2 m. For carrying objects, we want to use robot 2 in the warehouse environment in Fig 3. We want a motion planner to compute a motion model that can be used for multiple planning problems in this warehouse.

(i) Which sampling based motion planner would you choose and why? **[4 marks]**

(ii) Provide a detailed example of the result of this motion planner in the form of a graph. Explain its nodes and edges and how they are obtained. **[13 marks]**

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.